Summer School - From kinetic equations to statistical mechanics

28th June – 2rd July 2021, Saint Jean de Monts

Monday 28th June 2021

14:00-15:30 Nils Berglund (Université d'Orléans) : Long-time dynamics of stochastic differential equations

15:30-16:00 Break

16:00-17:30 Jean-Marc Bouclet (Université Toulouse III - Paul Sabatier) : Introduction to pseudo-differential calculus

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Tuesday 29th June 2021

9:00-10:30 Nils Berglund (Université d'Orléans) : Long-time dynamics of stochastic differential equations

10:30-11:00 Break

11:00-12:30 Jean-Marc Bouclet (Université Toulouse III - Paul Sabatier) : Introduction to pseudo-differential calculus

12:30-14:00 Lunch break

14:00-15:30 François Golse (École Polytechnique) : Kinetic Models: From Newtons equations to collisionless kinetic models

15:30-16:00 Break

16:00-17:00 Antoine Lejay (INRIA Nancy Grand-Est) : Simuler des EDS : schémas à pas exponentiel

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Wednesday 30th June 2021

9:00-10:30 Nils Berglund (Université d'Orléans) : Long-time dynamics of stochastic differential equations

10:30-11:00 Break

11:00-12:30 François Golse (École Polytechnique) : Kinetic Models: Examples of collisional kinetic models

12:30-14:00 Lunch break

Thursday 1st July 2021

9:00-10:30 Jean-Marc Bouclet (Université Toulouse III - Paul Sabatier) : Introduction to pseudo-differential calculus

10:30-11:00 Break

11:00-12:00 Jean-François Bony (CNRS, Université de Bordeaux) : Asymptotiques spectrales d'opérateurs de Fokker-Planck.

12:00-14:00 Lunch break

14:00-15:30 François Golse (École Polytechnique) : Kinetic Models: The regularity problem for the Landau equation

15:30-16:00 Break

16:00-17:00 - Marjolaine Puel (Université Côte d'Azur) : Approximation diffusion pour quelques équations cinétiques.

Friday 2nd July 2021

9:00-10:00 Gabriel Stoltz (École des Ponts ParisTech) : Computational statistical physics and hypocoercivity

10:00-10:30 Break

10:30-11:30 Françoise Pène (Université de Bretagne Occidentale) : Probabilistic limit theorems for chaotic dynamical systems, some results for dispersive billiards and Lorentz gases

11:30-12:30 Armen Shirikyan (CY Cergy Paris Université) : A mathematical introduction to the fluctuation theorem in non-equilibrium statistical mechanics

12:30-14:00 Lunch break

Abstracts

Courses

- **N. Berglund, Long-time dynamics of stochastic differential equations**

  This series of three lectures will present an overview of some tools that are useful to characterise the long-time behaviour of stochastic differential equations (SDEs) in $\mathbb{R}^d$.
  Lecture 1 : SDEs and PDEs
  Brownian motion, Ito's formula, SDEs, infinitesimal generator, Dynkin's formula, Feynman-Kac formula, Fokker-Planck equation.
  Lecture 2 : Invariant measures for SDEs
  This lecture will present a number of methods allowing to prove existence and uniqueness of invariant measures for SDEs, as well as to obtain results on speed of convergence towards the invariant measure.
  Lecture 3 : Large deviations
  Introduction to large-deviation principles, Schilder's theorem for scaled Brownian motion, Freidlin-Wentzell theory for SDEs with small noise, applications to the exit problem.

- **J.-M. Bouclet, Introduction to pseudo-differential calculus** The purpose of this course, targeted to non experts, is to introduce basic results of pseudo-differential calculus (symbol classes, symbolic calculus, L^2 bounds). We'll illustrate it with applications, in particular arising in kinetic equations.

- **F. Golse: Kinetic Models**
  Lecture 1: From Newtons equations to collisionless kinetic models
  In this first lecture, we explain how the Vlasov equation is derived from Newtons second law of motion written for each element in alarge system of

identical point particles interacting via a pairwise potential with Lipschitz continuous gradient. This analysis uses the notion of Wasserstein distance between Borel probability measures on the Euclidean space (Dobrushin, 1979).

We shall also discuss the case where the particle interaction involves the Coulomb potential, which is singular at the origin. In this case, we explain how the Euler-Poisson system which is equivalent to the Vlasov-Poisson system for monokinetic solutions can be derived from the N particle dynamics in the large N limit (Serfaty-Duerinckx 2020)

Lecture 2: Examples of collisional kinetic models

The second lecture in this course presents two examples of collision integrals in kinetic theory, namely the Boltzmann equation (used in the kinetic theory of neutral gases), and the Landau equation (used in the kinetic theory of charged particles). For each one of these two models, we present the basic properties of the collision integrals (weak formulation, conservation laws, H theorem, equilibrium distribution functions)

Lecture 3: The regularity problem for the Landau equation

The third and last lecture is focussed on the regularity of solutions to the Landau equation with Coulomb interaction.

In the first part of this lecture, we recall the famous De Giorgi method (1957), which provided the missing piece leading to the resolution of Hilberts 19th problem (on the analycity of extremals of regular variational problems).

In the second part of this lecture, we explain how this method can be applied to investigate the regularity problem for the space-homogeneous Landau equation with Coulomb interaction.

Conferences

- **J.-F. Bony, Asymptotiques spectrales d'opérateurs de Fokker-Planck.** La première partie de l'exposé sera consacrée à la construction des quasimodes BKW ainsi qu'à leur rôle dans le calcul du spectre d'opérateur de Schrödinger. Dans la seconde partie, on donnera l'asymptotique des petites valeurs propres du laplacien de Witten. Pour cela, on utilisera des quasimodes gaussiens qui sont adaptés à ces opérateurs et évitent de passer par le complexe de Witten. Travail en collaboration avec D. Le Peutrec et L. Michel.
- **A. Lejay, Simuler des EDS : schémas à pas exponentiel**

  Les technique de simulation des équations différentielles stochastiques (EDS) telles que le schéma dEuler, se justifient communément par des techniques de calcul stochastique. Il est néanmoins possible de considérer des techniques de simulation qui sappuient sur lanalyse des équations aux dérivées partielles sous-jacentes (Par exemple, au mouvement brownien est associé lopérateur de Laplace). Alors que nous pouvons interpréter le schéma dEuler pour les EDS comme relié à une approximation de la densité de transition (noyau parabolique), nous présenterons dans cet exposé comment on peut tirer profit de la connaissance du noyau de Green pour créer des schémas à pas exponentiel. Nous expliquerons lutilité de cette approche pour traiter des situations où les coefficients sont discontinus.

- **F. Pène, Probabilistic limit theorems for chaotic dynamical systems, some results for dispersive billiards and Lorentz gases**
  After recalling the classical probabilistic limit theorems for sums of independent identically distributed random variables, we will consider analogous results in a dynamical context. Motivated by examples coming from statistical mechanics, we will be interested in the Sinai billiards and in the periodic Lorentz gas. We will also consider the Bunimovich stadium billiard and dispersive billiards with cusps. All these billiards are chaotic, but enjoy different kinds of behaviour. This talk will end with a focus on an important analytic tool hidden behind several limit theorems mentioned in this talk: the regularity of the spectrum and eigenprojectors of operators.

- **M. Puel, Approximation diffusion pour quelques équations cinétiques.**

  Le but de cet exposé est de présenter la méthode dapproximation de la solution dune équation cinétique par un équilibre thermodynamique dont la densité satisfait une équation de diffusion. Nous considérerons des modèles collisionnels linéaires comme l'équation de Boltzmannn linéaires et l'équation de Fokker Planck. Je présenterai plusieurs méthodes qui reposent sur les différentes notions abordées lors de cette master class.

- **A. Shirikyan, A mathematical introduction to the fluctuation theorem in non-equilibrium statistical mechanics**
  We present a general framework of the theory of entropic fluctuations for deterministic and stochastic systems. After introducing some simple objects related to the entropy production, we show that the fluctuation relation as proposed by Evans-Searles, Gallavotti-Cohen, and Lebowitz-Spohn is a consequence of the large deviations principle (LDP). We next discuss in detail the case of finite-state Markov chains and show how the general theory works for this simplest situation. Finally, we study a class of chaotic dynamical systems for which we prove the validity of LDP with a convex good rate function satisfying the level-3 fluctuation relation.

- **Gabriel Stoltz, Computational statistical physics and hypocoercivity**
  I will provide an introduction to molecular dynamics, the computational implementation of the theory of statistical physics. The discussion will be

focused on the properties of Langevin dynamics, a degenerate stochastic differential equation, which can be seen as a perturbation of Hamiltonian dynamics. From an analytical point of view, the generator of Langevin dynamics is a degenerate elliptic operator. The evolution of the law of the stochastic process is governed by the Fokker-Planck equation, and its longtime convergence can be obtained via hypocoercive techniques, which I will review. I will also present the implication of these analytical results in terms of error estimates for the computation of average properties of molecular systems by estimating the asymptotic variance of time averages in a central limit theorem