

# A backward stochastic excursion with Ying HU Abstracts

1. RAINER BUCKDAHN (Université de Bretagne Occidentale)

Title: Mean field stochastic control under sublinear expectation

Abstract: Our work is devoted to the study of Pontryagin's stochastic maximum principle for a mean-field optimal control problem under Peng's G-expectation. The dynamics of the controlled state process is given by a stochastic differential equation driven by a G-Brownian motion, whose coefficients depend not only on the control, the controlled state process but also on its law under the G-expectation. Also the associated cost functional is of mean-field type. Under the assumption of a convex control state space we study the stochastic maximum principle, which gives a necessary optimality condition for control processes. Under additional convexity assumptions on the Hamiltonian it is shown that this necessary condition is also a sufficient one. The main difficulty which we have to overcome in our work consists in the differentiation of the G-expectation of parameterized random variables. As particularly delicate it turns out to handle with the G-expectation. For this we have to study a measurable selection theorem for set-valued functions whose values are subsets of the representing set of probability measures for the G-expectation.

2. PIERRE CARDALIAGUET (Université Paris-Dauphine)

Title: On the convergence rates for mean field control problems.

Abstract: In this work with J. Jackson, N. Mimikos-Stamatopoulos and P. Souganidis, we investigate the rate at which optimal control problems of a large number Nof "particles" converge to a mean field control problem. In a previous paper with P. Souganidis, we provided a global, algebraic, convergence rate for this problem. A later article by S. Daudin, F. Delarue and J. Jackson improved the convergence rate but also showed that such an algebraic convergence rate is actually optimal. Here we prove here that there exists an open, dense region of regularity of the value function of the limit problem on which the convergence rate is linear. We also show that the optimal feedback converges (with a rate) in this region and obtain a concentration inequality for optimal trajectories of the N-particle problem. 3. JEAN-FRANÇOIS CHASSAGNEUX (Université Paris cité)

Title: Computing the stationary measure of McKean-Vlasov SDEs

Abstract: Under some confluence assumption, it is known that the stationary distribution of a McKean-Vlasov SDE is the limit of the empirical measure of its associated self-interacting diffusion. Our numerical method consists in introducing the Euler scheme with decreasing step size of this self-interacting diffusion and seeing its empirical measure as the approximation of the stationary distribution of the original McKean-Vlasov SDEs. This simple approach is successful (under some raisonnable assumptions...) as we are able to prove convergence with a rate for the Wasserstein distance between the two measures both in the L2 and almost sure sense. In this talk, I will first explain the rationale behind this approach and then I will discuss the various convergence results we have obtained so far.

This is a joint work with G. Pagès (Sorbonne Université)

#### 4. PAUL-ERIC CHAUDRU DE RAYNAL (Nantes Université)

Title: Smoothing effect of the Kolmogorov Equation on Wasserstein Space.

Abstract: The semigroup of the solution of a non-degenerate linear SDE (in the sense of McKean) inherits the smoothing properties of the heat semigroup. Such properties have allowed, among other things, to prove the well-posedness of the SDE and to obtain quantitative error estimates of its time discretization in general frameworks. The aim of this presentation is to exhibit similar properties for the semigroup generated by the solution of a non-degenerate McKean-Vlasov type SDE. The approach is based on a thorough study of the associated Kolmogorov equation, set on the Wasserstein space.

This talk is based on joint work with N. Frikha.

5. FRANÇOIS DELARUE (Université Côte d'Azur)

Title: Major/minor MFG: common noise helps

Abstract: The purpose of the talk is to revisit conditions under which major/minor MFGs have a unique equilibrium over time intervals of arbitrary length. To this end, it is assumed that the costs to the minor players are monotone and that the intensity of the noise (referred to as 'common') driving the dynamics of the major player is large enough with respect to some characteristic quantities of the game. These conditions allow us to construct a classical solution to the corresponding master equation. Joint work with Chenchen Mou (City University of Hong-Kong).

6. SHENGJUN FAN (China University of Mining and Technology)

Title: Multi-dimensional backward stochastic differential equations of diagonally quadratic generators: the general result

Abstract: The present talk is devoted to a general solvability of a multi-dimensional backward stochastic differential equation (BSDE) of a diagonally quadratic generator, by relaxing the assumptions of Hu and Tang (2016, SPA) on the generator and terminal value. Three new results are established on the local and global solutions when the terminal value is bounded and the generator is subject to some general assumptions. When the terminal value is unbounded but is of exponential moments of arbitrary order, an existence and uniqueness result is given, which seems to be the first general solvability of system of quadratic BSDEs with unbounded terminal values. This generalizes and strengthens some existing results via some new ideas. This a joint work with Prof. Ying Hu and Shanjian Tang.

7. MONIQUE JEANBLANC (Université d'Evry)

Title: Shinkage of filtrations and applications to BSDE.

Abstract: In this talk, we study two nested filtrations : a Brownian filtration and a larger one. We assume that there exists a martingale in the large filtration which enjoy predictable representation property and that the Brownian motion ia a semimartingale in the large filtration. Then, we study the projection of a semimartingale in the large filtration on the Brownian filtration. We apply our results to BSDE and produce an extention of the paper of Claudia Ceci, Alessandra Cretarola, and Francesco Russo (BSDEs Under Partial information and Financial Applications, SPA 124, 2628 2643,2014)

8. JUAN LI (Shandong University)

Title: Path-dependent controlled Mean-Field coupled forward-backward SDEs. The associated stochastic maximum principle

Abstract: In the present paper we discuss a new type of mean-field coupled forwardbackward stochastic differential equations (MFFBSDEs). The novelty consists in the fact that the coefficients of both the forward as well as the backward SDEs depend not only on the controlled solution processes  $(X_t, Y_t, Z_t)$  at the current time t, but also on the law of the paths of (X, Y, u) of the solution process and the process by which it is controlled. The existence for such a MFFBSDE which is fully coupled through the law of the paths of (X, Y) in the coefficients of both the forward and the backward equations is proved under rather general assumptions. The main part of the work is devoted to the study of Pontryagin's maximal principle for such a MFFBSDE. The dependence of the coefficients of the law of the paths of the solution processes and their control makes that a completely new and interesting criterion for the optimality of a stochastic control for the MFFBSDE is obtained. Furthermore, we show that this necessary optimality condition is, under the assumption of convexity of the Hamiltonian, also sufficient. The talk is based on joint work with Rainer Buckdahn (UBO, France), Junsong Li (SDU, China), Chuanzhi Xing (SDU, China)

9. GECHUN LIANG (University of Warwick)

Title: Forward performance processes with unbounded market price of risk.

Abstract: In this talk, we discuss forward performance processes in the homothetic case, such as power and exponential cases, in an incomplete market with unbounded market price of risk. The corresponding representation in terms of ergodic BSDE turns out to be quadratic in z with z unbounded, so the ergodic BSDE is genuinely quadratic in z with unbounded coefficients. We apply the technique of De Giorgi type iteration to deal with unbounded coefficients. These estimates turn out to be crucial for solving the ergodic BSDE and constructing optimal trading strategies for forward performance processes. This is based on joint work with Adrien Richou.

10. YIQING LIN (Shanghai Jiao Tong University)

Title: Reflected BSDE driven by a marked point process with a convex/concave generator

Abstract: In this paper, a class of reflected backward stochastic differential equations (RBSDE) driven by a marked point process (MPP) with a convex/concave generator is studied. Based on fixed point argument,  $\theta$ -method and truncation technique, the well-posedness of this kind of RBSDE with unbounded terminal condition and obstacle is investigated. Besides, we present an application on the pricing of American options via utility maximization, which is solved by constructing an RBSDE with a convex generator.

11. JIN MA (University of Southern California)

Title: A Forward-backward View of Ying Hu

Abstract: In this talk I give a brief, and by no means complete, description of Ying Hu's career.

12. MARIE-AMÉLIE MORLAIS (Le Mans Université)

Title: Optimal switching problems with an infinite set of modes: an approach by randomization and constrained backward SDEs

Abstract: We address a general optimal switching problem over finite horizon for a stochastic system described by a differential equation driven by Brownian motion. The main novelty is the fact that we allow for infinitely many modes (or regimes), i.e. the possible values of the piecewise-constant control process. One specific feature is that all the given coefficients in the model are allowed to be path-dependent. The main aim is to introduce a suitable (scalar) backward stochastic differential equation (BSDE), with a constraint on the martingale part, that allows to give a probabilistic representation of the value function of the given problem. This is achieved by randomization of control, i.e. by introducing an auxiliary optimization problem which has the same value as the starting optimal switching problem and for which the desired BSDE representation is obtained. This is a joint work with Marco Fuhrman.

## 13. ÉTIENNE PARDOUX (Aix-Marseille Université)

Title: Recent results on epidemic models

Abstract: In 1927, two Scottish epidemiologists, Kermack and McKendrick, published a paper on a SIR epidemic model, where each infectious individual has an age of infection dependent infectivity, and a random infectious period whose law is very general. This paper was quoted a huge number of times, but almost all authors who quoted it considered the simple case of a constant infectivity, and a duration of infection following the exponential distribution, in which case the integral equation model of Kermack and McKendrick reduces to an ODE. It is classical that an ODE epidemic model is the Law of Large Numbers limits, as the size of the population tends to infinity, of finite population stochastic Markovian epidemic models. One of our main contributions in recent years has been to show that the integral equation epidemic model of Kermack and McKendrick is the law of large numbers limit of stochastic non Markovian epidemic models. It is not surprising that the model of Kermack and McKendrick, unlike ODE models, has a memory, like non Markovian stochastic processes. One can also write the model as a PDE, where the additional variable is the age of infection of each infected individual. Similar PDE models have been introduced by Kermack and McKendrick in their 1932 and 1933 papers, where they add a progressive loss of immunity. We have also shown that this 1932-33 model is the Law of Large Numbers limit of appropriate finite population non Markovian models. Joint work with R. Forien (INRAE Avignon, France), G. Pang (Rice Univ., Houston, Texas, USA) and A.B. Zotsa-Ngoufack (AMU and Univ. Yaoundé 1)

#### 14. SHIGE PENG (Shandong University)

Title: Nonlinear Space-time white noise under probability model uncertainty

Abstract: In most practical cases, the probability measure of systems themself is impossible to be obtained. This type of higher level uncertainty can be described by a family of probability measures in which we are unable to determine the "true one". In this situation, the notion of nonlinear expectation is often used to obtain a robust expectation. A typical probabilistic model is to use G-Brownian motion in the place of classical Brownian motion. But Brownian has a strong character of time indication which is difficult to be used as space-indexed probability model. In this talk we provide a new type of space-type and space-time-type white noise and the corresponding stochastic PDE driving by this noise. This a joint work with Xiaojun Ji.

15. Agnès Sulem (INRIA Paris)

Title: Graphon mean-field backward stochastic differential equations with jumps and associated dynamic risk measures

Abstract: We study backward stochastic differential equations with jumps (BS-DEs) with mean-field type heterogeneous interactions governed by graphons, and associated dynamic risk measures. We prove existence, uniqueness and measurability results of solutions of these systems under some regularity assumptions, and provide moment estimates. Under additional conditions, comparison theorems are obtained. Continuity and stability of such systems are also established. We then prove convergence of finite interacting mean-field particle systems with heterogeneous interactions to graphon mean-field BSDEs. Finally, we introduce the graphon dynamic risk measure induced by the solution of a graphon mean-field BSDE system and study its properties. In particular, a dual representation theorem is provided in the convex case. Joint work with Hamed Amini (University of Florida) and Zhongyuan Cao (NYU Shangai).

16. SHANJIAN TANG (Fudan University)

Title: Multidimensional Backward Stochastic Differential Equations With Rough Drifts

Abstract: In this talk, we address a multidimensional backward stochastic differential equation (BSDE) with an additional rough drift (rough BSDE), and give the existence and uniqueness of the adapted solution, either when the terminal value and the geometric rough path are small, or when each component of the rough drift only depends on the corresponding component of the first unknown variable (but we drop the one-dimensional assumption of Diehl and Friz [Ann. Probab. 40 (2012), 1715-1758]). We also introduce a new notion of the p-rough stochastic integral for  $p \in [2, 3)$ , and then succeed in giving—through a fixed-point argument—a general existence and uniqueness result on a multidimensional rough BSDE with a general square-integrable terminal value, allowing the rough drift to be random and time-varying but having to be linear; furthermore, we connect it to a system of rough partial differential equations. The talk is based on my joint work with Jiahao Liang, which is to appear in Transaction of AMS.

# 17. GIANMARIO TESSITORE (Università degli Studi di Milano)

Title: Nonlinear random perturbations of PDEs depending on a small parameter

Abstract: In this study, we investigate a class of quasi-linear parabolic equations defined on a separable Hilbert space, with a small parameter in front of the non-linear second-order term. Our primary focus is on the asymptotic behaviour of the solutions as the small parameter vanishes. We demonstrate that a large deviations principle applies and explicitly characterise the associated action functional. The nonlinear SPDE is approached using either direct probabilistic methods or analytical techniques. This work extends Freidlin and Koralov's (PTRF 2010) results to an infinite-dimensional setting. Joint work with Sandra Cerrai (University of Maryland) and Giuseppina Guatteri (Politecnico di Milano).

18. ZUOQUAN XU (Hong Kong Polytechnic University)

Title: Multidimensional Indefinite Stochastic Riccati Equations and Zero-Sum Linear-Quadratic Stochastic Differential Games with Non-Markovian Regime Switching

Abstract: This paper is concerned with a two-player zero-sum linear-quadratic stochastic differential game in a regime switching model. The coefficients of the controlled inhomogeneous linear system depend on the underlying noises, so it is a non-Markovian regime switching model. Based on a new kind of multidimensional indefinite stochastic Riccati equation (SRE) and a multidimensional linear backward stochastic differential equation (BSDE) with unbounded coefficients, we can provide optimal feedback control-strategy pairs for the two players in a closed-loop form. The main contribution of this paper, which is of great importance in its own right from the BSDE theory point of view, is to prove the existence and uniqueness of the solution to the new kind of multidimensional indefinite SRE. Notably, the first component of the solution (as a process) is capable of taking positive, zero, and negative values simultaneously. We also obtain the corresponding optimal feedback control-strategy pairs for homogeneous system under closed convex cone control constraint. Finally, these results are applied to portfolio selection games with full or partial no-shorting constraint in a regime switching market with random coefficients. This talk is based on a joint work with Panpan Zhang, Shandong University.

19. FALEI WANG (Shandong University)

Title: Quadratic Mean-Field Reflected BSDEs

Abstract: In this paper, we analyze mean-field reflected backward stochastic differential equations when the driver has quadratic growth in the second unknown z. Using a linearization technique and the BMO martingale theory, we first apply a fixed-point argument to establish the uniqueness and existence result for the case with bounded terminal condition and obstacle. Then, we develop a successive approximation procedure to remove the boundedness condition on the terminal condition and obstacle when the generator is concave (or convex) with respect to the second unknown. In a similar way, we also consider mean reflected backward stochastic differential equations. Based on a joint work with Y. Hu and R. Moreau.

## 20. XUNYU ZHOU (Columbia University)

Title: Reinforcement Learning in Continuous Time

Abstract: This talk will highlight the latest development on theory and algorithms for reinforcement learning in continuous time with continuous state space and possibly continuous control space, including entropy regularized exploratory formulation, policy evaluation, policy gradient, and q-learning.